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THE STRESS-STRAIN CURVE MODEL IN THE FORM OF AN EXTREMAL OF A NON-INTEGRABLE LINEAR VARIATION FORM

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The article develops an idea that the stress-strain curve for an arbitrary material is the extremum of some functional. However, for irreversible processes, the using of the principle of stationarity of some functional is incorrect, because due to the dissipation of the deformation process, the possible work of internal forces is non-integrable. Therefore, it is proposed to use the generalized variational principle of L. I. Sedov for modeling the stress-strain curve of elastoplastic materials. A concept of sequential inclusion of certain deformation mechanisms on different segment of the stress-strain curve is proposed. According to this concept, each section of the stress-strain curve must correspond either to the stationary value of the corresponding functional, or to the stationary value of the non-integrated form of variations of the corresponding stress derivatives. The combination of naturally obtained spectra of boundary conditions at the ends of each segment leads to a variation-consistent formulation of the system of boundary and contact conditions of solutions of different differential equations on each segment of stress-strain curve. As a result, it is possible to construct a differentiable stress-strain curve over the entire area of the stress-strain curve definition. The resulting solution, in contrast to the Ramberg – Osgood empirical law, has a strictly liner segment. The obtained mathematical model was tested on experimental data of materials for various industrial purposes. The achieved accuracy of the mathematical model is sufficient for engineering applications.

Keywords: Ramberg – Osgood law; empirical stress-strain curves; stress-strain curve as a solution of the ordinary differential equation of the fourth order; stress-strain curves as an extreme of functional; processing of experimental data.

Introduction

Several empirical models have formulated in the literature that describe the stress-strain curve. One of the most popular models is the Ramberg – Osgood model [1]. This model is popular among scientists involved in modeling the properties of materials [2 – 21] and among engineers who solve problems of structural design of plastic materials [22 – 29].

There are two approaches to modeling the properties of elastoplastic materials. The first approach is the compilation of universal curves defined by one formula in the entire range of strains [1, 4 – 9, 30].

The second approach is to formulate the stress-strain curve as a multilink spline with two [10 – 18], three [19 – 21], or four [15] segments.

The Ramberg – Osgood law corresponds to the first approach.

It was shown [11] that the empirical Ramberg – Osgood law has two significant drawbacks limiting its use, both in modeling the properties of materials and in the design of structures from them. First, the modulus of the tangent to the stress-strain curve corresponding to the Ramberg – Osgood law for an engineering curve cannot take on a value of zero at the point of ultimate strength. Therefore, this law is incompatible with the condition of theoretical strength. Secondly, according to the Ramberg – Osgood law, the tangent module is a monotonically decreasing function, and therefore the stress-strain curve has no linear segment.

1. In [11] an alternative empirical model was proposed, which is not defined on the segment $0 \le \varepsilon' \le 1$, but on the segment $\varepsilon_e^* \le \varepsilon^* \le 1$. On the segment $0 \le \varepsilon^* \le \varepsilon_e^*$ postulated strictly linear law:

$$\sigma = \begin{cases} E_e^* \varepsilon^*, 0 \le \varepsilon^* \le \varepsilon_e^* \\ E_e^* \varepsilon^* - (E_e^* - 1)\left(\dfrac{\varepsilon^* - \varepsilon_e^*}{1 - \varepsilon_e^*}\right)^{\eta}, \varepsilon_e^* \le \varepsilon^* \le 1. \end{cases} \quad (1)$$

Here $\sigma^* = \sigma/\sigma_c$, $\varepsilon^* = \varepsilon/\varepsilon_c$ and $(\varepsilon_c{}^*; \sigma_c{}^*)$ — are the coordinates of the point of ultimate strength of the material on the stress-strain curve. This alternative empirical model is equivalent to the stress-strain curve must be divided into two fundamentally different segments, separated by a characteristic point for each material — point, which is called "proportionality limit" and has coordinates $(\varepsilon_e^*; \sigma_e^*)$. The first segment of the stress-strain curve — strictly linear. The constant modulus of elasticity $E_e^* = \varepsilon_e^*/\sigma_e^*$ and the parameter $\eta = (1 - \varepsilon_e^*)/(1 - \varepsilon_e^*/\sigma_e^*)$ can be determined through the coordinates of a point called the "proportional limit."

2. Developing this idea, it was assumed in [17] that a solution of some ordinary fourth-order differential equation can be used on a nonlinear segment, since the stress itself and its derivative (the tangent modulus to the stress-strain curve) must be specified at the ends of the nonlinear segment:

$$\varepsilon^{*2}\sigma^{*''''} + 4\varepsilon^*\sigma^{*'''} + (2 - \eta)\sigma^{*''} = 0. \tag{2}$$

The solution of (2) gives:

$$\sigma^*(\varepsilon^*) = \begin{cases} C_0 + C_1\varepsilon^* + C_2\varepsilon^{*n_2} + C_3\varepsilon^{*n_3}, 0 \le \varepsilon^* \le \varepsilon_e^* \\ c_0 + c_1\varepsilon^* + c_2\varepsilon^{*n_2} + c_3\varepsilon^{*n_3}, \varepsilon_e^* \le \varepsilon^* \le 1. \end{cases} \tag{3}$$

The boundary conditions (4) – (5) are:

$$\begin{cases} \sigma^*(0) = 0 \\ \sigma^{*\prime}(0) = E_e^* \end{cases}, \begin{cases} \sigma^*(\varepsilon_e^*) = \sigma_e^* \\ \sigma^{*\prime}(\varepsilon_e^*) = E_e^* \end{cases}, 0 \le \varepsilon^* \le \varepsilon_e^*, \tag{4}$$

$$\begin{cases} \sigma^*(\varepsilon_e^*) = \sigma_e^* \\ \sigma^{*\prime}(\varepsilon_e^*) = E_e^* \end{cases}, \begin{cases} \sigma^*(1) = 1 \\ \sigma^{*\prime}(1) = 0 \end{cases}, \varepsilon_e^* \le \varepsilon^* \le 1. \tag{5}$$

Satisfying the boundary conditions and substituting in (3) one can obtain the stress-strain curve.

3. The next step in stress-strain curve mathematic modelling is the idea, that there is some functional exist, the stationarity value of which will give not only a kinetic equation for stress, but a variation-coordinated spectrum of boundary conditions on each segment of stress-strain curve. In [29] it has been shown, that on different segments desired functional has a different number of summands defining different "deformation mechanisms." As the result, each new "deformation mechanism" change the structure or order of differential equation on the current segment. For nonlinear-elastic materials, such functional has the form:

$$U = \frac{1}{2}\int_0^{\varepsilon_e^*} A_{11}\sigma^{*\prime}\sigma^{*\prime}d\varepsilon^* + \frac{1}{2}\int_{\varepsilon_e^*}^{\varepsilon_c^*} [A_{22}\varepsilon^{*2}\sigma^{*''}\sigma^{*''} + $$

$$+ 2A_{21}\varepsilon^*\sigma^{*''}\sigma^{*\prime} + A_{11}\sigma^{*\prime}\sigma^{*\prime}]d\varepsilon^*. \tag{6}$$

The summand $A_{22}\varepsilon^{*2}\sigma^{*''}\sigma^{*''}$, which is included on the second segment of the stress-strain curve and continues to act up to failure, defines the "second stress-derived square" mechanism.

The summand $2A_{21}\varepsilon^*\sigma^{*''}\sigma^{*\prime}$ defines a "bilinear on the second and first stress-derived" mechanism, which includes on the second segment of the stress-strain curve simultaneously with the "quadratic" one and continues to act further.

The summand $A_{11}\sigma^{*\prime}\sigma^{*\prime}$ defines the only deformation mechanism acting on the first segment and corresponding to the linear Hook's law equation. It does not "turn off" and continues to operate on the second segment. Naturally, the parameter value A_{11} defining this mechanism must have the same value throughout the segments of stress-strain curve on which this mechanism act. The requirement of stationarity of functional (6) gives

$$\delta U = 0. \tag{7}$$

Unlike the previous approach, on different segments of stress-strain curve the curve defined by different kinetic equations. Really, on segment of linearity the kinetic equality is

$$\sigma^{*''} = 0. \tag{8}$$

On segment of nonlinearity the kinetic equality is

$$\varepsilon^{*2}\sigma^{*''''} + 4\varepsilon^*\sigma^{*'''} + (2 - \eta)\sigma^{*''} = 0. \tag{9}$$

Here η — physical parameter, reflecting mechanical properties of the material and connecting with parameters A_{22}, A_{21}, A_{11}. The variation principle (7) gives a consistent system of boundary conditions and conjugation conditions for solutions of kinetic equations (8) and (9).

4. However, for irreversible processes, the using of the stationarity principle of some functional is not correct, because, due to the dissipation of the deformation process, the possible work of internal forces is non-integrable. The non-linear segment should divided into two segments.

On the first segment there is no dissipation and the deformation processes reversible, but nonlinear. In the second section, the dissipation process starts and deformations become irreversible and nonlinear. Both parts separated by specific point of material $(\varepsilon_r{}^*; \sigma_r{}^*)$. This point will called the "reversibility limit." Really, before this point stress-strain curve describes reversible process of deforming. If process of deforming pass through this point, it becomes irreversible. That is, the process of dissipation is "turned on" behind the "reversibility limit" point $(\varepsilon_r{}^*; \sigma_r{}^*)$, and some sort of dissipation process start to act.

This article dedicated to the realization of this idea.

Formulation of dissipative model as principle of stationarity of non-integrated linear variation form

In [22], a generalization of the L. I. Sedov variation equation for modeling irreversible processes was proposed. The essence of generalization is that Sedov's variation equation represented as the sum of variation of the functional of the reversible part plus the set of dissipation channels. The simplest of non-integrated linear variation form called the "dissipation channel." Its arguments formed by one of the bilinear terms in the functional of the reversible part. In the present case of the reversible part of the functional, there is only a single dissipation channel can be:

$$\overline{\sigma U_{21}} = \int_{\varepsilon_r^*}^{\varepsilon_c^*} B_{21}\varepsilon^*(\sigma^{*\prime\prime}\delta\sigma^{*\prime} - \sigma^{*\prime}\delta\sigma^{*\prime\prime})d\varepsilon^*. \quad (10)$$

Behind the point $(\varepsilon_r^*;\sigma_r^*)$, the variation principle of stationarity of functional (7) becomes incorrect and replaced by the variation principle of stationarity of the non-integrated variation form:

$$\delta U + \overline{\sigma U_{21}} = 0. \quad (11)$$

This variation principle can simulate stress-strain curve of elastoplastic materials. We follow the concept of sequential inclusion of various deformation mechanisms on different segments of the stress-strain curve. According to this theory, the stress-strain curve will divided into three segments: linear reversible segment $0 \le \varepsilon^* \le \varepsilon_e^*$, nonlinear reversible segment $\varepsilon_e^* \le \varepsilon^* \le \varepsilon_r^*$ and nonlinear irreversible segment $\varepsilon_r^* \le \varepsilon^* \le \varepsilon_c^*$.

Constructing stress-strain curve as a conjunction problem for three solutions

Linear reversible segment $0 \le \varepsilon^* \le \varepsilon_e^*$. Modeling the stress-strain curve with the simplest quadratic functional, we obtain a linear strain model:

$$U = \frac{1}{2}\int_0^{\varepsilon_e^*} A_{11}\sigma^{*\prime}\sigma^{*\prime}d\varepsilon^*. \quad (12)$$

Here A_{11} — a physical parameter reflecting the mechanical properties of the first deformation mechanism.

The stationarity condition of (12) gives the kinetic equation of the stress-strain curve as well as the natural boundary conditions:

$$\delta U = \int_0^{\varepsilon_e^*} A_{11}\sigma^{*\prime}\delta\sigma^{*\prime}d\varepsilon^* = \int_0^{\varepsilon_e^*} -A_{11}\sigma^{*\prime\prime}\delta\sigma^* d\varepsilon^* +$$

$$+ A_{11}\sigma^{*\prime}\delta\sigma^*\Big|_0^{\varepsilon_e^*} = 0. \quad (13)$$

Kinetic equation, follows from (13):

$$\sigma^{*\prime\prime} = 0. \quad (14)$$

The solution to kinetic equation (14) is as follows:

$$\sigma^* = C_0 + C_1\varepsilon^*. \quad (15)$$

According to (13), assuming that stresses are set at the ends of the segment (stresses variations are zero), we obtain:

$$\begin{cases}\sigma^*(0) = 0\\ \sigma^*(\varepsilon_e^*) = \sigma_e^*\end{cases} \Rightarrow \begin{cases}C_0 = 0\\ C_1 = E_e^*\end{cases}. \quad (16)$$

Here $E_e^* = \sigma_e^*/\varepsilon_e^*$ — dimensionless Young's modulus; $\varepsilon_e^*, \sigma_e^*$ — dimensionless coordinates of a point of proportionality limit on a stress-strain curve.

Linear Hooke's law on a stress-strain curve on segment $0 \le \varepsilon^* \le \varepsilon_e^*$ as a result received:

$$\sigma^* = E_e^*\varepsilon^*. \quad (17)$$

Nonlinear reversible segment $\varepsilon_e^* \le \varepsilon^* \le \varepsilon_r^*$. As already noted in the introduction, on a nonlinear segment, the differential equation must be a fourth-order equation. Accordingly, an additional component containing the square of the second stress derivative should appear in the functional. We will treat the appearance/disappearance of the additional deformation mechanism in the functional as "on/off."

When passing through the proportional limit point, on the second section of the stress-strain curve, the simultaneous activation of two new deformation mechanisms postulated, and the functional becomes:

$$U = \frac{1}{2}\int_{\varepsilon_e^*}^{\varepsilon_r^*}[A_{22}\varepsilon^{*2}\sigma^{*\prime\prime}\sigma^{*\prime\prime} + 2A_{21}\varepsilon^*\sigma^{*\prime\prime}\sigma^{*\prime} +$$

$$+ A_{11}\sigma^{*\prime}\sigma^{*\prime}]d\varepsilon^*. \quad (18)$$

The deformation mechanism, determined by physical parameter $A_{22}\sigma^{*\prime\prime}\sigma^{*\prime\prime}$, which is start to act on the second segment of the stress-strain curve and continues to act further, is defined by the "second stress-derived square."

The deformation mechanism, determined by physical parameter $A_{21}\sigma^{*\prime\prime}\sigma^{*\prime}$, defines a "bilinear on the second and first stress-derived" deformation mechanism, which start to act on the second segment simultaneously with the "quadratic" one and continues to act further.

The only deformation mechanism acting on the first segment and corresponding to the linear

Hook's law equation, does not "turn off" and continues to act on the second segment. Otherwise, the functional (18) would not positively defined, and the corresponding solution would not be the only one. Naturally, the parameter A_{11} value, defining this mechanism, must have the same value throughout the segments of stress-strain curve on which this mechanism act.

The variation equation on the second segment of the stress-strain curve is:

$$\delta U = \int_{\varepsilon_e}^{\varepsilon_r} [A_{22}\varepsilon^2\sigma'''' + 4A_{22}\varepsilon\sigma''' +$$

$$+(2A_{22} + A_{21} - A_{11})\sigma'']\delta\sigma d\varepsilon +$$

$$+(A_{22}\varepsilon^2\sigma'' + A_{21}\varepsilon\sigma')\delta\sigma'\Big|_{\varepsilon_e}^{\varepsilon_r} - [A_{22}\varepsilon^2\sigma''' + 2A_{22}\varepsilon\sigma'' +$$

$$+(A_{21} - A_{11})\sigma']\delta\sigma\Big|_{\varepsilon_e}^{\varepsilon_r} = 0. \quad (19)$$

Kinetic equation:

$$\varepsilon^{*2}\sigma^{*''''} + 4\varepsilon^*\sigma^{*'''} + (2 - \eta)\sigma^{*''} = 0. \quad (20)$$

Material parameter

$$\eta = \frac{A_{11} - A_{21}}{A_{22}}. \quad (21)$$

The solving of kinetic equation (20), taking into account (21), is as follows:

$$\sigma^*(\varepsilon^*) = c_0 + c_1\varepsilon^* + c_2\varepsilon^{*n_2} + c_3\varepsilon^{*n_3}. \quad (22)$$

According to (19), assuming that stresses are set at the ends of the segment (stresses variations are zero) and tangent modulus are set in addition (stresses derivative variations are zero), we obtain:

the boundary conditions for solution on nonlinearity reversible segment when $\varepsilon^* = \varepsilon_e^*$

$$\begin{cases} \sigma^*(\varepsilon_e^*) = c_0 + c_1\varepsilon_e^* + c_2\varepsilon_e^{*n2} + c_3\varepsilon_e^{*n3} = \sigma_e^* \\ \sigma^{*\prime}(\varepsilon_e^*) = c_1 + c_2 n_2\varepsilon_e^{*n2-1} + c_3 n_3\varepsilon_e^{*n3-1} = E_e^*; \end{cases} \quad (23)$$

the boundary conditions for solution on nonlinearity reversible segment when $\varepsilon^* = \varepsilon_r^*$

$$\begin{cases} \sigma^*(\varepsilon_r^*) = c_0 + c_1\varepsilon_r^* + c_2\varepsilon_r^{*n2} + c_3\varepsilon_r^{*n3} = \sigma_r^* \\ \sigma^{*\prime}(\varepsilon_r^*) = c_1 + c_2 n_2\varepsilon_r^{*n2-1} + c_3 n_3\varepsilon_r^{*n3-1} = E_r^*. \end{cases} \quad (24)$$

Nonlinear irreversible segment $\varepsilon_r^* \le \varepsilon^* \le \varepsilon_c^*$. As already noted in the introduction, that on a nonlinear interval two segments must exist. The first, described above, defines the deformation process throughout is reversible. The second should take into account irreversible deformation processes, which determines the plasticity property. This means, that when crossing the reversibility limit point, a new, dissipative deformation mechanism (10), turned on. At the same time, all previous mechanisms also continue to act. A generalization of the L. I. Sedov variation equation becomes as (11). Taking in account the structures (10) and (19), the stationarity requirement of this non-integrated variation form (11) yields the following variation equation:

$$\delta U + \overline{\delta U_{21}} = \int_{\varepsilon_r^*}^{\varepsilon_c^*} [A_{22}\varepsilon^{*2}\sigma^{*''''} + (4A_{22} - 2B_{21})\varepsilon^*\sigma^{*'''} +$$

$$+(2A_{22} + A_{21} - A_{11} - 3B_{21})\sigma^{*''}]\delta\sigma^* d\varepsilon^* +$$

$$+[A_{22}\varepsilon^{*2}\sigma^{*''} + (A_{21} - B_{21})\varepsilon^*\sigma^{*\prime}]\delta\sigma^{*\prime}\Big|_{\varepsilon_r^*}^{\varepsilon_c^*} -$$

$$-[A_{22}\varepsilon^{*2}\sigma^{*'''} + 2(A_{22} - B_{21})\varepsilon^*\sigma^{*''} +$$

$$+(A_{21} - A_{11} - B_{21})\sigma^{*\prime}]\delta\sigma^*\Big|_{\varepsilon_r^*}^{\varepsilon_c^*} = 0. \quad (25)$$

Together with the already introduced parameter (21), we introduce a new physical parameter of the material

$$\xi = B_{21}/A_{22}. \quad (26)$$

Kinetic equation, follows from (25):

$$\varepsilon^{*2}\sigma^{*''''} + (4 - 2\xi)\varepsilon^*\sigma^{*'''} + (2 - \eta - 3\xi)\sigma^{*''} = 0. \quad (27)$$

The solving of kinetic equation (27), taking into account (21) and (26), is follow:

$$\sigma^*(\varepsilon^*) = a_0 + a_1\varepsilon^* + a_2\varepsilon^{*n_4} + a_3\varepsilon^{*n_5}. \quad (28)$$

According to (25), assuming that stresses are set at the ends of the segment (stresses variations are zero) and tangent modulus are set in addition (stresses derivative variations are zero), we obtain:

the boundary conditions for solution on nonlinearity irreversible segment when $\varepsilon^* = \varepsilon_r^*$

$$\begin{cases} \sigma^*(\varepsilon_r^*) = a_0 + a_1\varepsilon_r^* + a_2\varepsilon_r^{*n_4} + a_3\varepsilon_r^{*n_5} = \sigma_r^* \\ \sigma^{*\prime}(\varepsilon_r^*) = a_1 + a_2 n_4\varepsilon_r^{*n_4-1} + a_3 n_5\varepsilon_r^{*n_5-1} = E_r^*; \end{cases} \quad (29)$$

the boundary conditions for solution on nonlinearity irreversible segment when $\varepsilon^* = \varepsilon_c^* = 1$

$$\begin{cases} \sigma^*(1) = a_0 + a_1 + a_2 + a_3 = \sigma_c^* \\ \sigma^{*\prime}(1) = a_1 + a_2 n_4 + a_3 n_5 = E_c^*. \end{cases} \quad (30)$$

Ten parameters C_0; C_1; c_0; c_1; c_2; c_3; a_0; a_1; a_2; a_3 are determined from the solution of the problem of conjugation of the stress-strain curve at the contact points (16), (23), (24), (29) and (30). Thereafter, the stress-strain curve can plotted from known

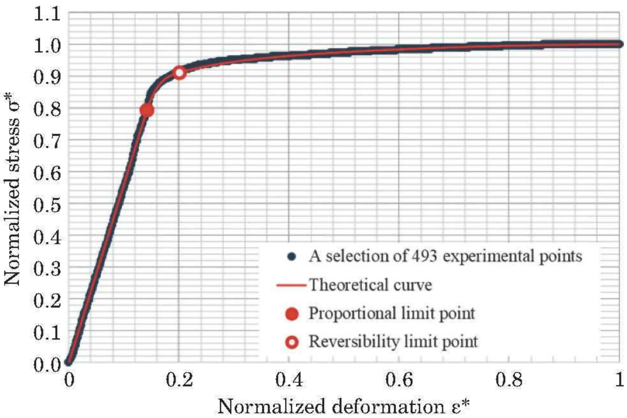

Fig. 1. Theoretical stress-strain curve and experimental data for 30CrMnSiNi2A armor steel (16532 CSN): $\varepsilon_e^* = 0.14199$, $\sigma_e^* = 0.79231$; $\varepsilon_r^* = 0.20081$, $\sigma_r^* = 0.91154$; $E_0^* = 5.60$; $S = 0.0048$

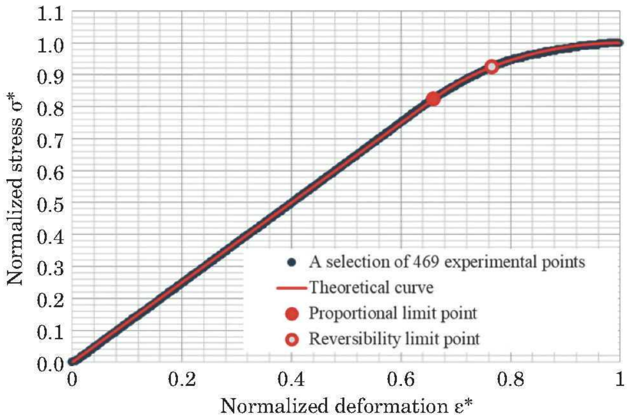

Fig. 2. Theoretical stress-strain curve and experimental data for 40Cr2Ni2MA armor steel (4340 ASTM): $\varepsilon_e^* = 0.65885$, $\sigma_e^* = 0.82437$; $\varepsilon_r^* = 0.76546$, $\sigma_r^* = 0.92473$; $E_0^* = 1.25$; $S = 0.0033$

physical parameters or determine these parameters using a sample of experimental points

$$\sigma^* = \begin{cases} C_0 + C_1\varepsilon^* \text{ for } 0 \le \varepsilon^* \le \varepsilon_e^* \\ c_0 + c_1\varepsilon^* + c_2\varepsilon^{*n_2} + c_3\varepsilon^{*n_3} \text{ for } \varepsilon_e^* \le \varepsilon^* \le \varepsilon_r^* \\ a_0 + a_1\varepsilon^* + a_2\varepsilon^{*n_4} + a_3\varepsilon^{*n_5} \text{ for } \varepsilon_r^* \le \varepsilon^* \le \varepsilon_c^*. \end{cases} \quad (31)$$

Tangent modulus:

$$E^* = \sigma^{*\prime} =$$

$$= \begin{cases} C_1 \text{ for } 0 \le \varepsilon^* \le \varepsilon_e^* \\ c_1 + c_2 n_2 \varepsilon^{*n_2 - 1} + c_3 n_3 \varepsilon^{*n_3 - 1} \text{ for } \varepsilon_e^* \le \varepsilon^* \le \varepsilon_r^* \\ a_1 + a_2 n_4 \varepsilon^{*n_4 - 1} + a_3 n_5 \varepsilon^{*n_5 - 1} \text{ for } \varepsilon_r^* \le \varepsilon^* \le \varepsilon_c^*. \end{cases} \quad (32)$$

Formally, physical parameters, determining mechanical properties of elastoplastic material, are coordinates of three characteristic points of stress-strain curve $(\varepsilon_e^*; \sigma_e^*)$, $(\varepsilon_r{}^*; \sigma_r{}^*)$, $(\varepsilon_c^*; \varepsilon_c^*)$, as well as parameters, characterizing acting deformation mechanisms A_{22}, A_{21}, A_{11}, B_{21}:

$$\begin{cases} C_i = C_i(\varepsilon_e^*, \sigma_e^*) \\ c_i = c_i(\varepsilon_e^*, \sigma_e^*, \varepsilon_r^*, \sigma_r^*, E_r^*, E_c^*, n_2(\eta), n_3(\eta)) \\ a_i = a_i(\varepsilon_r^*, \sigma_r^*, E_r^*, E_c^*, n_4(\xi, \eta), n_5(\xi, \eta)). \end{cases} \quad (33)$$

However, in the model under consideration, due to the normalization, the absolute values of the coordinates of the ultimate strength point $(\varepsilon_c^*; \sigma_c)$ are not included in the curve equation. In addition, between four parameters A_{22}, A_{21}, A_{11}, B_{21}, only two their linear combinations ξ, η are included in the curve equation.

As a result, the constructed theoretical curve (31) is an eight-parameter curve:

$$\begin{cases} C_i = C_i(\varepsilon_e^*, \sigma_e^*) \\ c_i = c_i(\varepsilon_e^*, \sigma_e^*, \varepsilon_r^*, \sigma_r^*, E_r^*, E_c^*, \eta) \\ a_i = a_i(\varepsilon_r^*, \sigma_r^*, E_r^*, E_c^*, \eta, \xi). \end{cases} \quad (34)$$

Special attention should paid to the parameter E_r^* that is most likely to be associated with other physical parameters, by analogy with $E_e^* = \sigma_e^*/\varepsilon_e^*$. Moreover, it may be possible to formulate three additional restrictions, either local or integral, that allow as to express physical parameters through the coordinates of the curve's feature points. As a result, the number of physical parameters will determined solely by the number of characteristic points on the curve and the values of the tangent moduli at these points.

Thus, the result obtained suggests that, in general, all properties of elastoplastic materials determined by the geometry of the stress-strain curve. The proof of this hypothesis will be the subject of further research.

Methodology for processing experimental data

There is used a Gradient Descent Method to processing the experimental data, based on a numerical search for the minimum sum of the quadratic deviations theoretical stress-strain curve as a function of seven parameters on a finite number of sample data points [12, 14, 17].

The analysis of the predictive power of the considered theoretical model carried out on materials from four groups, two materials from each group. Armor steels were chosen 30CrMnSiN2A (16532 CSN) and 40Cr2Ni2MA (4340 ASTM); aerospace alloys D16A (2024 USA) and BT6 (6Al-4V Grade5);

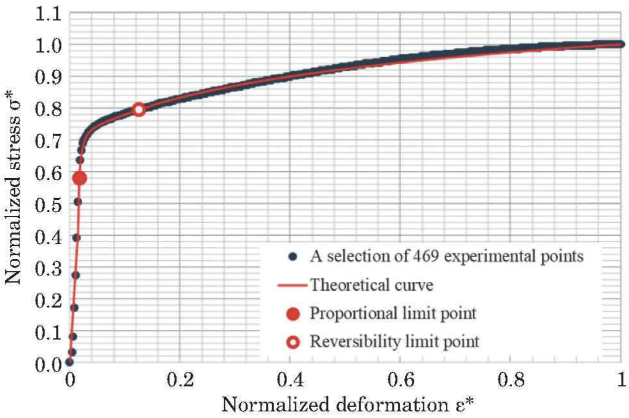

Fig. 3. Theoretical stress-strain curve and experimental data for aluminum alloy D16 (AA2024 USA/ANSI H35.2): $\varepsilon_e^* = 0.01706$, $\sigma_e^* = 0.57955$; $\varepsilon_r^* = 0.12580$, $\sigma_r^* = 0.79545$; $E_0^* = 28.00$; $S = 0.0105$

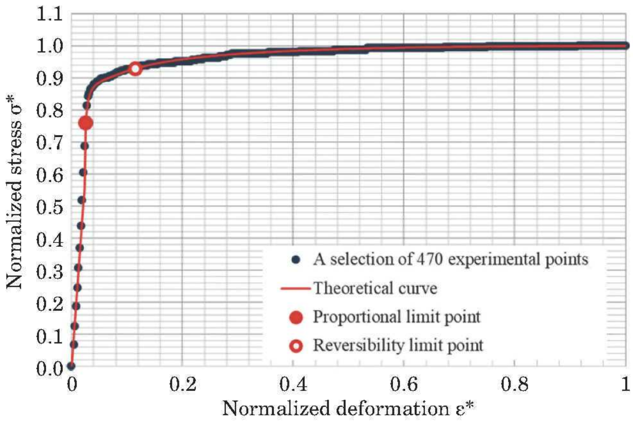

Fig. 6. Theoretical stress-strain curve and experimental data for pipeline steel 20ХГР (1.5526 DIN): $\varepsilon_e^* = 0.02553$, $\sigma_e^* = 0.76033$; $\varepsilon_r^* = 0.11489$, $\sigma_r^* = 0.92975$; $E_0^* = 24.00$; $S = 0.0089$

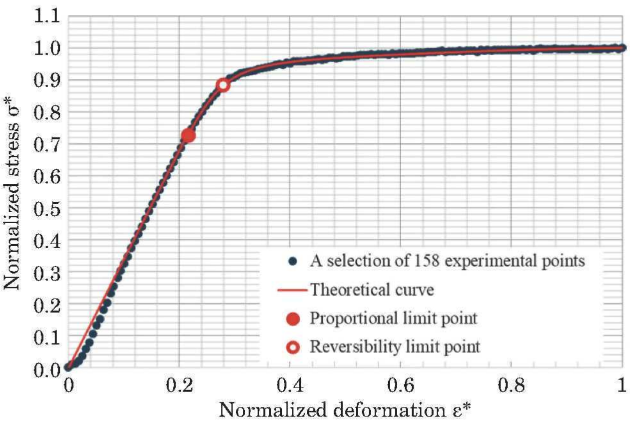

Fig. 4. Theoretical stress-strain curve and experimental data for VT6 titanium alloy (Ti-6Al-4V USA/AMS): $\varepsilon_e^* = 0.21592$, $\sigma_e^* = 0.72725$; $\varepsilon_r^* = 0.27963$, $\sigma_r^* = 0.88397$; $E_0^* = 3.40$; $S = 0.0123$

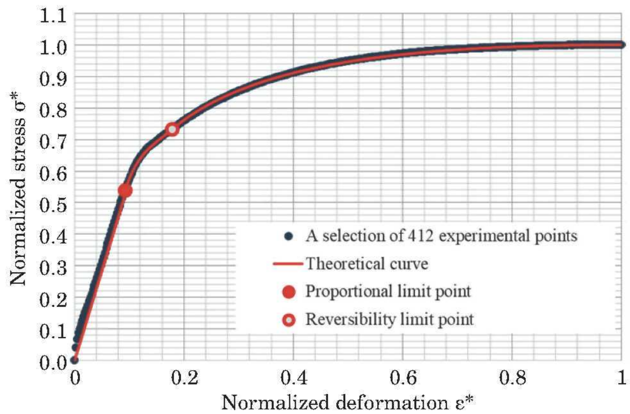

Fig. 7. Theoretical strain curve and experimental data for steel for general engineering St3sp (A414 GradeA): $\varepsilon_e^* = 0.09269$, $\sigma_e^* = 0.53903$; $\varepsilon_r^* = 0.17784$, $\sigma_r^* = 0.73209$; $E_0^* = 5.70$; $S = 0.0086$

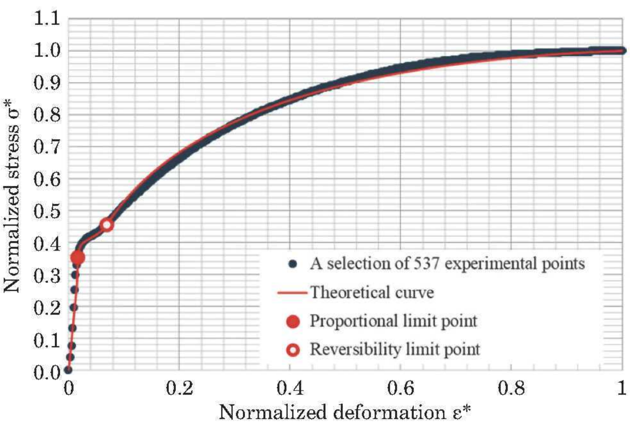

Fig. 5. Theoretical stress-strain curve and experimental data for pipeline steel 08X18H10 (304 USA/ASTM): $\varepsilon_e^* = 0.01676$, $\sigma_e^* = 0.35336$; $\varepsilon_r^* = 0.06890$, $\sigma_r^* = 0.45583$; $E_0^* = 18.00$; $S = 0.0124$

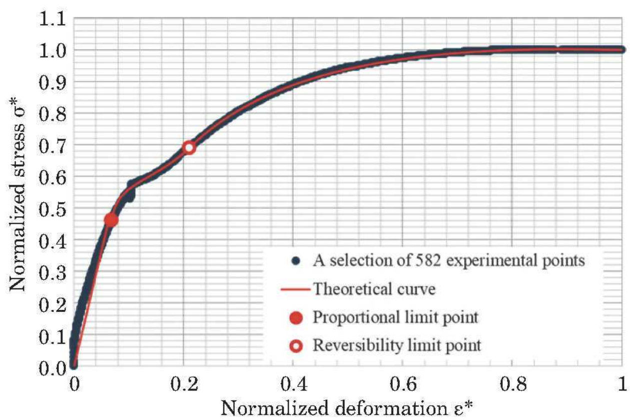

Fig. 8. Theoretical strain curve and experimental data for steel for general engineering 35 (A682 Grade 1035 USA/ASTM): $\varepsilon_e^* = 0.06750$, $\sigma_e^* = 0.46288$; $\varepsilon_r^* = 0.21000$, $\sigma_r^* = 0.68996$; $E_0^* = 7.30$; $S = 0.0278$

pipeline steels 08X18H10 (304 ASTM) and 20XGR (1.5526 DIN); steels for general engineering St3sp (A414 Grade A) and steel 35 (1035 ASTM).

Theoretical stress-strain curves construct in accordance with (31) after determining the physical parameters of materials (Figs. 1 – 8).

CONCLUSION

The article develops the idea that stress-strain curve is an extreme of some functional. According to the concept of activation of different deformation mechanisms on different sections of stress-strain curve, each segment of stress-strain curve must correspond to its functional. The naturally obtained spectrum of conjunction problems for these functionals leads to a variation-consistent formulation of the system of boundary and conjunction conditions of solutions to different differential equations on each segment of the stress-strain curve. This approach extends to dissipative deformation processes. In accordance with the generalization of L. I. Sedov, the variation of the functional on the nonlinear irreversible segment complemented by a non-integrable linear variation form that determines the dissipation process. The principle of stationarity of the functional replaced by a more general stationarity principle of non-integrable linear variation form. For verification, curves constructed for two types of armor steel, two aviation alloys, two pipe steels and two types of steel for general mechanical engineering. The standard deviation of the theoretical curve for samples of armor steels did not exceed 0.5 %, for aerospace alloys it turned out to be about 1 %, for pipe steels a little less than 1 %, for machine-building steel without a hardening zone less 1 %, and for machine-building steel — less than 3 %. The achieved accuracy of the mathematical model sufficient for engineering applications.

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